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LETTER TO THE EDITOR

Coherent Jaynes–Cummings dynamics modified by a quasi-instantaneous perturbation of the two-level system

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Abstract

The coherent Jaynes–Cummings (JC) dynamics modified by a quasi-instantaneous perturbation of the two-level system is discussed, and the corresponding evolution operator is determined with the help of the anticommutator analogue of the Baker–Hausdorff lemma. The evolution operator leads to a coherent superposition of two JC evolutions, one with the original values of the coupling constant and detuning, and the other with the modified, effective values of these parameters followed (in the resonant case) by a phase pulse. This leads to various possibilities for controlling and modifying the final state of the harmonic oscillator depending on the choice of the initial state and the perturbing phase and on the instant the perturbation acts.

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The dynamics of a single, two-level system which is coupled to a quantum harmonic oscillator is described by the Jaynes–Cummings (JC) model [1] and has been a subject of considerable interest (see, e.g. [2–7]). The model can be implemented in two representative scenarios in quantum optics, namely ion traps and cavity QED. In ion traps the centre-of-mass motion of the trapped ion is harmonic, and it couples to an internal atomic transition when the ion is irradiated by a laser [8]. In cavity QED, the harmonic oscillator is a mode of the quantized radiation field, coupled to a resonant electronic transition of atoms sent through the resonator and undergoing JC dynamics [9]. In quantum optics it has become possible to create quantum states of high purity [10]. In the view of potential applications, e.g. in a quantum computer, the coherent control and monitoring of such quantum states is becoming important. Here we discuss the coherent JC dynamics interrupted, at an intermediate instant, by a fast perturbation of the two-level system. In particular, with the help of the anticommutator analogue of the Baker–Hausdorff lemma, we determine the corresponding unitary evolution operator and find that it leads to a coherent superposition of two JC evolutions, one with the original values of the coupling constant and detuning, and the other with effective values of these parameters (that depend on the instant the perturbation acts) followed (in the resonant case) by a phase pulse. This offers various possibilities of modifying and controlling the final state of the harmonic oscillator.

Let us consider generally a nonresonant interaction between a two-level system, with lower state $|-\rangle$ and upper state $|+\rangle$, and a harmonic oscillator, with annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$, respectively. The eigenstates of the number operator $\hat{n} = \hat{a}^\dagger \hat{a}$ are denoted by $|n\rangle$ with $n = 0, 1, 2, \ldots$. It is assumed that the transition energy of the two-level system, ϵ, is different from the energy of the oscillator quanta, $\hbar\omega$, so that the quantity $\Delta\epsilon \equiv \epsilon - \hbar\omega$ (detuning) is generally different from zero. The JC dynamics is described by the Hamiltonian [11, 12]

$$\hat{H}_{\mathrm{JC}} = \hat{H}_0 + \hat{H}_{\mathrm{I}} \tag{1}$$

where $\hat{H}_0$ represents the free Hamiltonian

$$\hat{H}_0 \equiv \frac{\epsilon}{2}\big(\hat{\sigma}_z \otimes \hat{1}\big) + \hbar\omega\big(\hat{1}_{2\times 2} \otimes \hat{n}\big) \tag{2}$$

and $\hat{H}_{\mathrm{I}}$ the interaction term

$$\hat{H}_{\mathrm{I}} \equiv \lambda\big(\hat{\sigma}_+ \otimes \hat{a}\big) + \lambda^*\big(\hat{\sigma}_- \otimes \hat{a}^\dagger\big). \tag{3}$$

Here λ denotes the coupling constant, $\hat{\sigma}_\pm \equiv \frac{1}{2}\big(\hat{\sigma}_x \pm \mathrm{i}\hat{\sigma}_y\big)$, and $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$ denote the Pauli spin matrices. The composite system, the two-level system plus harmonic oscillator, evolves following this JC interaction for a total duration t. If at an intermediate instant, τ $(0 < \tau < t)$, the JC evolution is interrupted by a quasi-instantaneous perturbation of the two-level system of the form [13]

$$\hat{H}_{\mathrm{P}} \equiv \hbar\varphi\big(\hat{\sigma}_-\hat{\sigma}_+ \otimes \hat{1}\big)\delta(t-\tau) \tag{4}$$

the coherent dynamics is described by the Hamiltonian $\hat{H}(t) = \hat{H}_{\mathrm{JC}} + \hat{H}_{\mathrm{P}}$. In (4), φ represents a phase. After the duration t, the evolution operator for the whole process is

$$\hat{U}(t,0) = \hat{U}(t,\tau+0)\hat{U}(\tau+0,\tau-0)\hat{U}(\tau-0,0). \tag{5}$$

Since $\hat{H}_{\mathrm{P}} = 0$ for any $t \neq \tau$, one has

$$\hat{U}(\tau,0) \to \hat{U}_{\mathrm{JC}}(\tau,0) \equiv \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\hat{H}_{\mathrm{JC}}\tau} \qquad \hat{U}(t,\tau) \to \hat{U}_{\mathrm{JC}}(t,\tau) = \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\hat{H}_{\mathrm{JC}}(t-\tau)} \tag{6}$$

and also

$$\begin{aligned}\hat{U}(\tau+0,\tau-0) &= \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}\int_{\tau-0}^{\tau+0}\hat{H}(t)\,\mathrm{d}t} = \mathrm{e}^{-\mathrm{i}\varphi\left(\hat{\sigma}_-\hat{\sigma}_+\otimes\hat{1}\right)} \\ &= \mathrm{e}^{-\mathrm{i}\frac{\varphi}{2}}\left[\cos\frac{\varphi}{2}\big(\hat{1}_{2\times 2}\otimes\hat{1}\big) + \mathrm{i}\sin\frac{\varphi}{2}\,\big(\hat{\sigma}_z\otimes\hat{1}\big)\right]\end{aligned} \tag{7}$$

where in the last step one uses $\hat{\sigma}_-\hat{\sigma}_+ = \frac{1}{2}(\hat{1}_{2\times 2} - \hat{\sigma}_z)$. In the special case $\varphi = \pi$, equation (7) reduces to $\hat{\sigma}_z \otimes \hat{1}$. The evolution operator (5) becomes

$$\hat{U}(t,0) = \mathrm{e}^{-\mathrm{i}\frac{\varphi}{2}}\left[\cos\frac{\varphi}{2}\,\hat{U}_{\mathrm{JC}}(t,0) + \mathrm{i}\sin\frac{\varphi}{2}\,\hat{U}_{\mathrm{JC}}(t,\tau)\big(\hat{\sigma}_z\otimes\hat{1}\big)\hat{U}_{\mathrm{JC}}(\tau,0)\right]. \tag{8}$$

This sum of two terms leads to a coherent superposition of straight JC evolution from zero to t, with probability $\cos^2\frac{\varphi}{2}$, and two consecutive JC evolutions (from zero to τ and then from τ to t) interrupted, at $t = \tau$, by the unitary operator $\hat{\sigma}_z \otimes \hat{1}$. Now in the second term one would like to exchange the position of the two factors, $\hat{U}_{\mathrm{JC}}(t,\tau)$ and $\hat{\sigma}_z \otimes \hat{1}$, in order to obtain again an uninterrupted JC evolution on the entire interval $0 \to t$. Unfortunately this is not possible directly since the operator identity

$$[\hat{X}\otimes\hat{Y}, \hat{X}'\otimes\hat{Y}'] = [\hat{X},\hat{X}']\otimes\hat{Y}\hat{Y}' + \hat{X}'\hat{X}\otimes[\hat{Y},\hat{Y}'] \tag{9}$$

shows that $[\hat{H}_{\rm JC}, \hat{\sigma}_z \otimes \hat{1}] = -2\lambda\big(\hat{\sigma}_+ \otimes \hat{a}\big) + 2\lambda^*\big(\hat{\sigma}_- \otimes \hat{a}^\dagger\big) \neq 0$. Instead, one splits the JC Hamiltonian (1) into the sum of two commuting terms, $\hat{H}_{\rm JC} = \hat{A} + \hat{B}$, where [14]

$$\hat{A} \equiv \hbar\omega\big[\tfrac{1}{2}\big(\hat{\sigma}_z \otimes \hat{1}\big) + \hat{1}_{2\times 2} \otimes \hat{n}\big] \qquad \hat{B} \equiv \hat{H}_{\rm I} + \frac{\Delta\epsilon}{2}\hat{\sigma}_z \otimes \hat{1}. \tag{10}$$

One has $[\hat{A}, \hat{B}] = 0$, so the two observables, $\hat{A}$ and $\hat{B}$, form a complete set of compatible observables (their simultaneous eigenkets are the well known dressed states), alternative to the complete set comprising the two observables $\hat{\sigma}_z \otimes \hat{1}$ and $\hat{1}_{2\times 2} \otimes \hat{n}$ that provide the base kets $|\pm\rangle \otimes |n\rangle \equiv |\pm, n\rangle$. Additionally, $[\hat{A}, \hat{\sigma}_z \otimes \hat{1}] = 0$ but $[\hat{B}, \hat{\sigma}_z \otimes \hat{1}] \neq 0$. Therefore the similarity transformation occurring in the second term of equation (8)

$$\hat{U}_{\rm JC}(t,\tau)(\hat{\sigma}_z \otimes \hat{1})\hat{U}_{\rm JC}(\tau,0) = {\rm e}^{-\frac{\rm i}{\hbar}\hat{A}(t-\tau)}\left[{\rm e}^{-\frac{\rm i}{\hbar}\hat{B}(t-\tau)}(\hat{\sigma}_z \otimes \hat{1}){\rm e}^{\frac{\rm i}{\hbar}\hat{B}(t-\tau)}\right]{\rm e}^{-\frac{\rm i}{\hbar}\hat{B}t}{\rm e}^{-\frac{\rm i}{\hbar}\hat{A}\tau} \tag{11}$$

cannot be determined using the Baker–Hausdorff lemma. Fortunately, the *anti*commutator $\{\hat{H}_{\rm I}, \hat{\sigma}_z \otimes \hat{1}\}$ vanishes as can be seen using the operator identity

$$\{\hat{X} \otimes \hat{Y}, \hat{X}' \otimes \hat{Y}'\} = \{\hat{X}, \hat{X}'\} \otimes \hat{Y}\hat{Y}' - \hat{X}'\hat{X} \otimes [\hat{Y}, \hat{Y}']. \tag{12}$$

One obtains from (10)

$$\{\hat{B}, \hat{\sigma}_z \otimes \hat{1}\} = \Delta\epsilon\big(\hat{1}_{2\times 2} \otimes \hat{1}\big) \tag{13}$$

and the similarity transformation appearing in (11) can be established using the anticommutator analogue of the Baker–Hausdorff lemma [15]

$${\rm e}^{\hat{X}}\,\hat{Y}{\rm e}^{-\hat{X}} = \left(\hat{Y} + \{\hat{X}, \hat{Y}\} + \frac{1}{2!}\{\hat{X}, \{\hat{X}, \hat{Y}\}\} + \frac{1}{3!}\{\hat{X}, \{\hat{X}, \{\hat{X}, \hat{Y}\}\}\} + \cdots\right){\rm e}^{-2\hat{X}}. \tag{14}$$

One takes $\hat{X} \to (-{\rm i}/\hbar)\hat{B}(t-\tau)$ and $\hat{Y} \to \hat{\sigma}_z \otimes \hat{1}$. The repeated anticommutators follow from (13)

$$\{\hat{B}, \{\hat{B}, \ldots \{\hat{B}, \hat{\sigma}_z \otimes \hat{1}\underbrace{\}\ldots\}\}}_{k} = \Delta\epsilon(2\hat{B})^{k-1} \tag{15}$$

with $k = 1, 2, 3, \ldots$, and one finds

$${\rm e}^{-\frac{\rm i}{\hbar}\hat{B}(t-\tau)}\big(\hat{\sigma}_z \otimes \hat{1}\big){\rm e}^{\frac{\rm i}{\hbar}\hat{B}(t-\tau)} = \big[\hat{\sigma}_z \otimes \hat{1} + \Delta\epsilon\hat{\mathcal{B}}(t-\tau)\big]{\rm e}^{\frac{2{\rm i}}{\hbar}\hat{B}(t-\tau)} \tag{16}$$

where we introduced the operator

$$\hat{\mathcal{B}}(u) \equiv \sum_{k=1}^{\infty}\frac{(-{\rm i}u/\hbar)^k}{k!}\big(2\hat{B}\big)^{k-1} = \frac{1}{2}\big({\rm e}^{\frac{2{\rm i}}{\hbar}\hat{B}} - \hat{1}\big) \tag{17}$$

which obviously commutes with $\hat{A}$. The similarity transformation (11) becomes

$$\hat{U}_{\rm JC}(t,\tau)\big(\hat{\sigma}_z \otimes \hat{1}\big)\hat{U}_{\rm JC}(\tau,0) = \big[\hat{\sigma}_z \otimes \hat{1} + \Delta\epsilon\ \hat{\mathcal{B}}(t-\tau)\big]\exp\left(-\frac{\rm i}{\hbar}\left(\hat{A} + \frac{2\tau - t}{t}\hat{B}\right)t\right). \tag{18}$$

Here, $\hat{H}'_{\rm JC} \equiv \hat{A} + \frac{2\tau-t}{t}\hat{B}$ is the JC Hamiltonian with modified $\hat{B} \to \hat{B}' \equiv \frac{2\tau-t}{t}\hat{B}$. Put differently, the operator $\hat{B}'$ is the old $\hat{B}$ but with *effective values* of the coupling constant and detuning,

$$\lambda \to \lambda' \equiv \frac{2\tau - t}{t}\lambda \qquad \Delta\epsilon \to \Delta\epsilon' \equiv \frac{2\tau - t}{t}\Delta\epsilon \tag{19}$$

(see equations (3) and (10)) so that the usual JC evolution operator, $\hat{U}'_{\rm JC}(t,0) \equiv {\rm e}^{-\frac{\rm i}{\hbar}\hat{H}'_{\rm JC}t}$, appears on the right-hand side of equation (18), only this time with the effective values λ' and $\Delta\epsilon'$. Note that in the special case when the instant at which the fast perturbation acts is $\tau = t/2$ the effective values of the coupling constant and detuning vanish, $\lambda' = \Delta\epsilon' = 0$, so that $\hat{B}' = 0$, $\hat{H}'_{\mathrm{JC}} = \hat{A}$ and $\hat{U}'_{\mathrm{JC}}(2\tau, 0) \equiv \mathrm{e}^{-\frac{2\mathrm{i}}{\hbar}\hat{A}\tau}$. This last represents simply the evolution operator for the unitary evolution of the composite system with noninteracting components.

Finally, the evolution operator (8), describing coherent JC dynamics modified by a quasi-instantaneous perturbation of the two-level system, becomes

$$\hat{U}(t,0) = \mathrm{e}^{-\mathrm{i}\frac{\varphi}{2}} \left\{ \cos\frac{\varphi}{2}\, \hat{U}_{\mathrm{JC}}(t,0) + \mathrm{i}\sin\frac{\varphi}{2} \left[\hat{\sigma}_z \otimes \hat{1} + \Delta\epsilon\, \hat{\mathcal{B}}(t-\tau)\right] \hat{U}'_{\mathrm{JC}}(t,0) \right\}. \tag{20}$$

This evolution operator leads to a coherent superposition of JC evolution from zero to t, with probability $\cos^2\frac{\varphi}{2}$, and a JC evolution which is also uninterrupted on the entire interval $0 \to t$, but corresponding to the effective values of the coupling constant and detuning (19), and which is followed by the operator $\hat{\sigma}_z \otimes \hat{1} + \Delta\epsilon\, \hat{\mathcal{B}}(t-\tau)$. This enables one to use the standard results of the JC dynamics, which include description of for example quantum recurrence, collapse and revival of the atom–field dynamics [11] but with added complexity arising from the perturbation (4) leading to the coherent superposition occurring in (20). Here we discuss briefly some special cases.

In case of resonant interaction of the two-level system and a harmonic oscillator, there is no detuning ($\Delta\epsilon = 0$) and the term containing $\hat{\mathcal{B}}(t-\tau)$ is absent from equation (20). One has a coherent superposition of two different JC evolutions, one with the original coupling constant λ, and the other with the effective constant λ' followed by a phase pulse $\hat{\sigma}_z \otimes \hat{1}$ (a fast unitary operation applied to the two-level system that introduces a relative phase between the levels). The value of the phase φ determines the relative contribution of the terms occurring in equation (20). In the case when $\varphi = \pi$, only the second term survives:

$$\hat{U}(t,0) \longrightarrow \left[\hat{\sigma}_z \otimes \hat{1} + \Delta\epsilon\, \hat{\mathcal{B}}(t-\tau)\right] \hat{U}'_{\mathrm{JC}}(t,0) \tag{21}$$

so that one has only the modified JC evolution (the one with the effective values of the coupling constant and detuning, equation (19)), which is followed by the operator $\hat{\sigma}_z \otimes \hat{1} + \Delta\epsilon\, \hat{\mathcal{B}}(t-\tau)$. This last consists of the term describing the phase pulse and, in the case of nonvanishing detuning, the term containing $\hat{\mathcal{B}}(t-\tau)$, which introduces interference of the two members in (21). Another parameter that controls the relative importance of the terms of the coherent superposition in (20) is the time, τ, at which the quasi-instantaneous perturbation acts. It has already been mentioned that $\tau = t/2$ leads to vanishing values of the effective coupling constant and detuning, thus simplifying to the extreme the second JC evolution in (20). For example, for $\varphi = \pi$ the final state of the composite system matches the initial state (except for the irrelevant phase pulse); the final state is then just the freely evolved initial state. On the other hand, the value of τ close to zero increases the phase of the (complex) coupling constant by π, $\lambda' \approx \mathrm{e}^{\mathrm{i}\pi}\lambda$, and also effectively reverses the sense of detuning, $\Delta\epsilon' \approx -\Delta\epsilon$. In this case, the situation is as if the JC dynamics were time reversed. All this offers various possibilities of controlling and modifying the resulting, final state of the oscillator depending on the initial state of the composite system. This latter is usually uncorrelated, described generally by the density operator $\hat{\rho}(0) = |\pm\rangle\langle\pm| \otimes \hat{\rho}_{\mathrm{oscillator}}(0)$. Thus, the two-state system is initially in the upper(lower) state while the oscillator is in an arbitrary, pure or mixed, state $\hat{\rho}_{\mathrm{oscillator}}(0) = \sum_{n,m} \rho_{n,m} |n\rangle\langle m|$ (such as coherent, squeezed, thermal or nonclassical maximum-entropy state etc). The final state results then from the coherent evolution, $\hat{\rho}(t) = \hat{U}(t,0)\ \hat{\rho}(0)\ \hat{U}^{\dagger}(t,0)$, with $\hat{U}(t,0)$ given by equation (20), and this is determined employing the corresponding dressed state basis.

In summary, we have discussed the coherent JC dynamics modified by a quasi-instantaneous perturbation of the two-level system of the form (4). The corresponding evolution operator, (20), is determined with the help of the anticommutator analogue of the Baker–Hausdorff lemma. This evolution operator leads to a coherent superposition of two JC evolutions, one with the original values of the coupling constant and detuning, and the other with the effective values of these parameters, (19), followed (in the resonant case) by a phase pulse. This leads to various possibilities for controlling and modifying the final state of the harmonic oscillator depending on the choice of the initial state, the perturbing phase, φ, and on the instant, τ, when the fast perturbation acts.

References

[1] Jaynes E T and Cummings F W 1963 *Proc. IEEE* **51** 89
[2] Meschede D, Walther H and Müler G 1985 *Phys. Rev. Lett.* **54** 551
[3] Rempe G, Walther H and Klein N 1987 *Phys. Rev. Lett.* **58** 353
[4] Buzano C, Rasetti M G and Rastello M L 1989 *Phys. Rev. Lett.* **62** 137
[5] Meekhof D M, Monroe C, King B E, Itano W M and Wineland D J 1996 *Phys. Rev. Lett.* **76** 1796
[6] Brune M, Schmidt-Kaler F, Maali A, Dreyer J, Hagley E, Raimond J M and Haroche S 1996 *Phys. Rev. Lett.* **76** 1800
[7] Hyunchul Nha, Young-Tak Chough and Kyungwon An 2000 *Phys. Rev.* A **63** 010301(R)
[8] Wineland D J, Monroe C, Itano W M, Leibfried D, King B E and Meekhof D M 1997 *Preprint* arXiv quant-ph/9710025
[9] Raithel G, Wagner C, Walther H, Narducci L M and Scully M O 1994 *Advances in Atomic, Molecular and Optical Physics* ed P Berman (New York: Academic) Suppl 2
[10] Bouwmeaster D, Ekert A and Zeilinger A (ed) 2000 *The Physics of Quantum Information* (Berlin: Springer)
[11] Milloni P W and Singh S 1991 *Advances in Atomic, Molecular and Optical Physics* vol 28, ed D Bates and B Bederson (Boston: Academic) p 78
[12] Cohen-Tannoudji C, Dupont-Roc J and Grynberg G 1992 *Atom–Photon Interactions, Basic Processes and Applications* (New York: Wiley) ch VI
[13] Agarwal G S, Scully M O and Walther H 2001 *Phys. Rev. Lett.* **86** 4271
[14] Louisell W H 1964 *Radiation and Noise in Quantum Electronics* (New York: McGraw-Hill) ch V
[15] Mendaš I and Milutinović P 1989 *J. Phys. A: Math. Gen.* **22** L687